Introduction to Statistics and R

Descriptive Statistics - Part II

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Section 1

Recap

Recap

- data collection process (handedness questionnaire, age guessing demonstration)
- statistical graphics
  - stem-leaf plot: counting data
  - histogram (bin size: shape vs. detail): absolute and relative frequencies
  - scatter plot: relationships
- Numerical statistics: mean, error

Section 2

Learning Goals

Learning Goals

- Numerical summary statistics: variance, precision, standard deviation, quantiles
- Relation between numerical and graphical summaries
- Probability density functions

Section 3

Handedness Data Analysis

Handedness Data Analysis

Table 1: Complete collected data set from the handedness inventory

id	writing	drawing	throwing	scissors	toothbrush	knife	spoon	broom	match	box	calc	score	right	left
1	rr	rr	rr	rr	rr	rr	rr	rr	rr	rr	1	1.0000000	20	0
2	r	r	r	r	r	r	r	r	r	r	NA	1.0000000	10	0
3	rr	rr	r	rr	rr	rr	rr	rr	rr	rr	NA	1.0000000	19	0
4	rr	rr	rr	rr	rr	rr	rr	rr	rr	rr	NA	1.0000000	20	0
5	rr	rr	rr	rr	rr	rr	rr	rr	rr	rr	-1	1.0000000	20	0
6	rr	rr	rr	rr	lr	lr	rr	rr	lr	r	0.85 ~ 1	0.6842105	16	3
7	rr	rr	r	rr	rr	r	rr	r	ll	rr	0.8	0.7647059	15	2
8	r	r	r	r	r	r	r	r	lr	r	0.81	0.8181818	10	1
9	rr	rr	rr	rr	rr	rr	rr	NA	lr	rr	NA	0.8888889	17	1
10	r	r	r	r	r	r	r	r	lr	r	0.81	0.8181818	10	1
11	r	r	r	r	r	r	r	r	r	r	1	1.0000000	10	0
12	r	r	lr	r	lr	r	r	lr	lr	r	NA	0.4285714	10	4
13	lr	rr	lr	rr	rr	rr	r	r	r	r	0.75	0.7500000	14	2

Handedness Data Analysis

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id	writing	drawing	throwing	scissors	toothbrush	knife	spoon	broom	match	box	calc	score	right	left
1	rr	rr	rr	rr	rr	rr	rr	rr	rr	rr	1	1.0000000	20	0
2	r	r	r	r	r	r	r	r	r	r	NA	1.0000000	10	0
3	rr	rr	r	rr	rr	rr	rr	rr	rr	rr	NA	1.0000000	19	0
4	rr	rr	rr	rr	rr	rr	rr	rr	rr	rr	NA	1.0000000	20	0
5	rr	rr	rr	rr	rr	rr	rr	rr	rr	rr	-1	1.0000000	20	0
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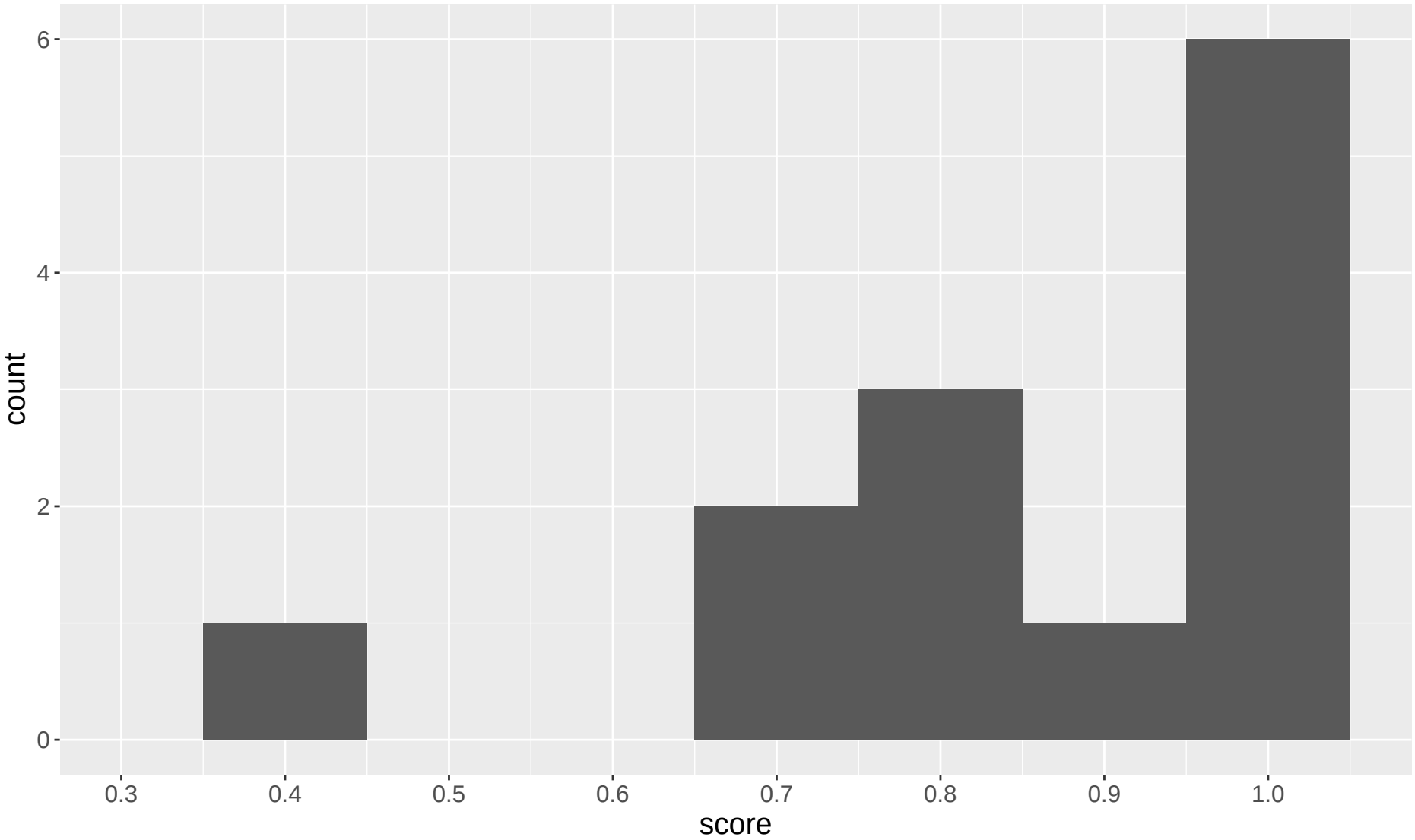
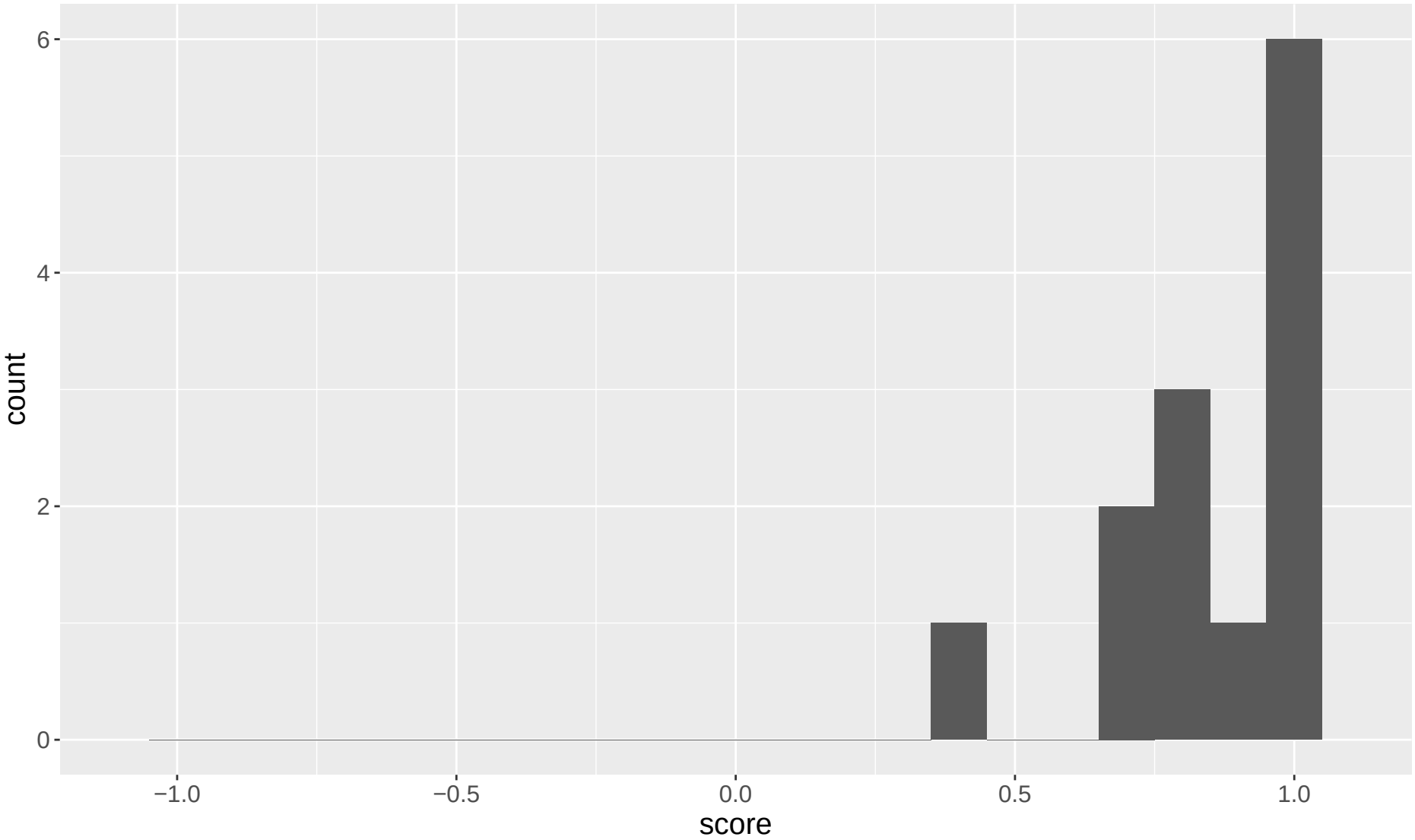
13	lr	rr	lr	rr	rr	rr	r	r	r	r	0.75	0.7500000	14	2

Handedness Data Analysis



Handedness Data Analysis



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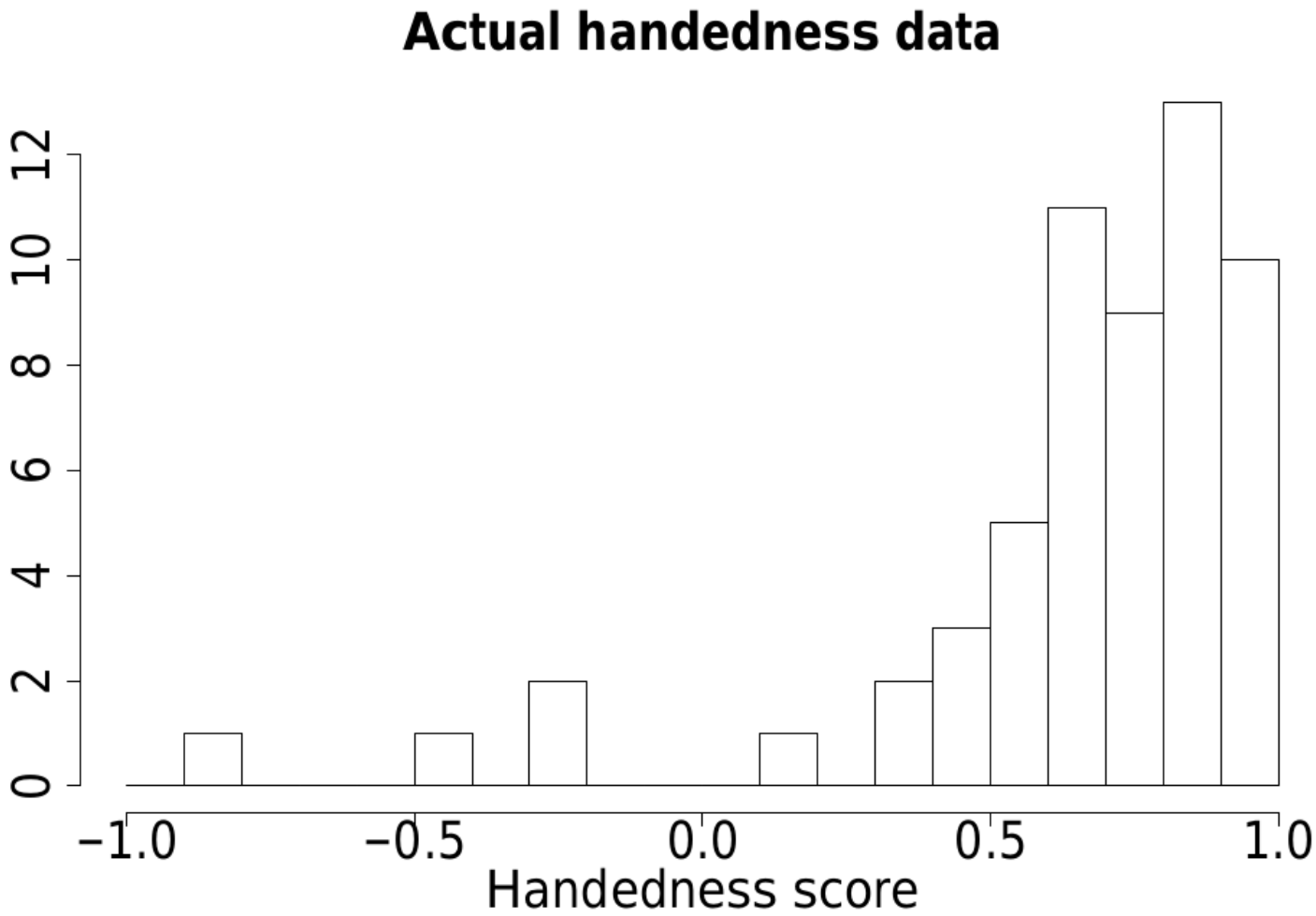
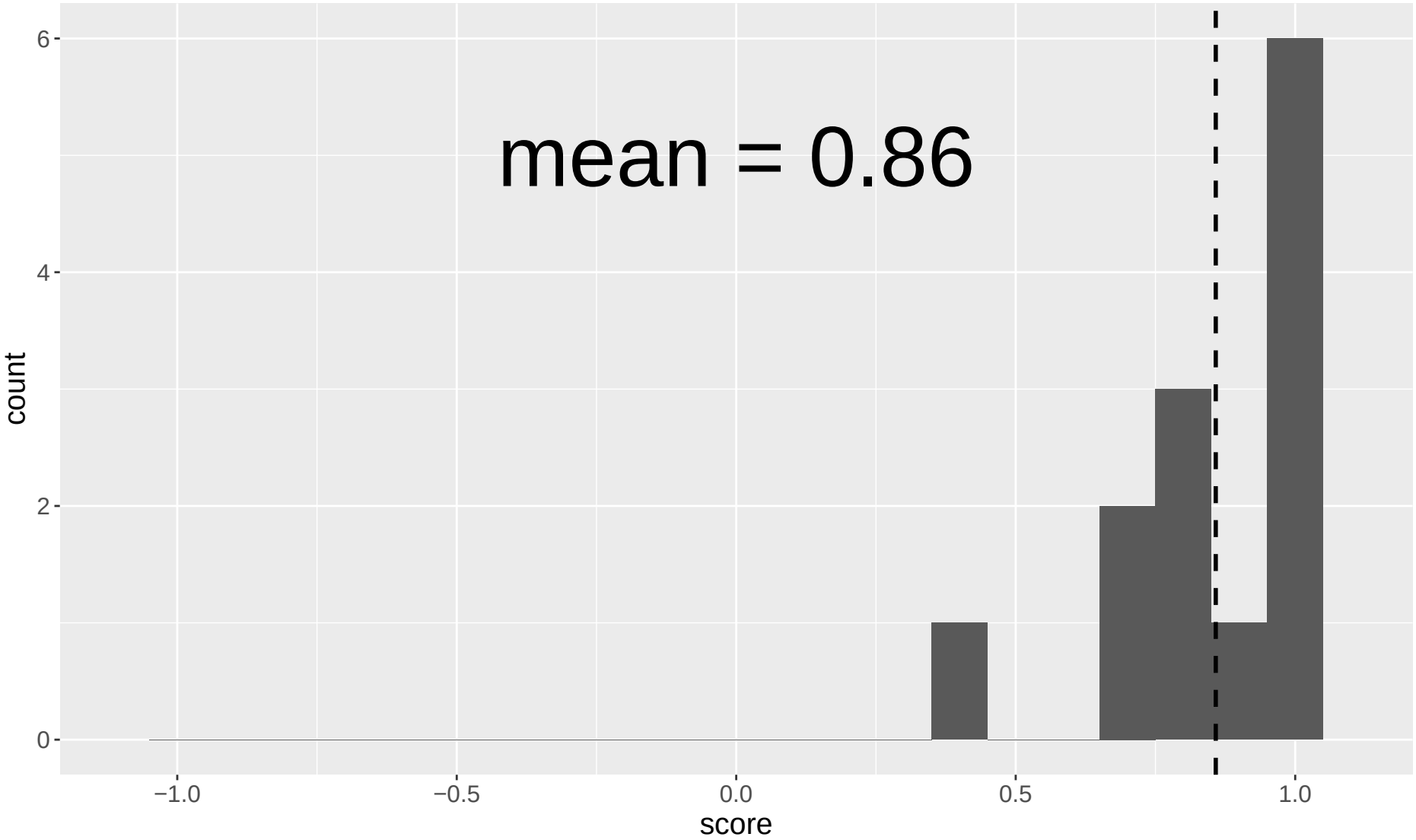
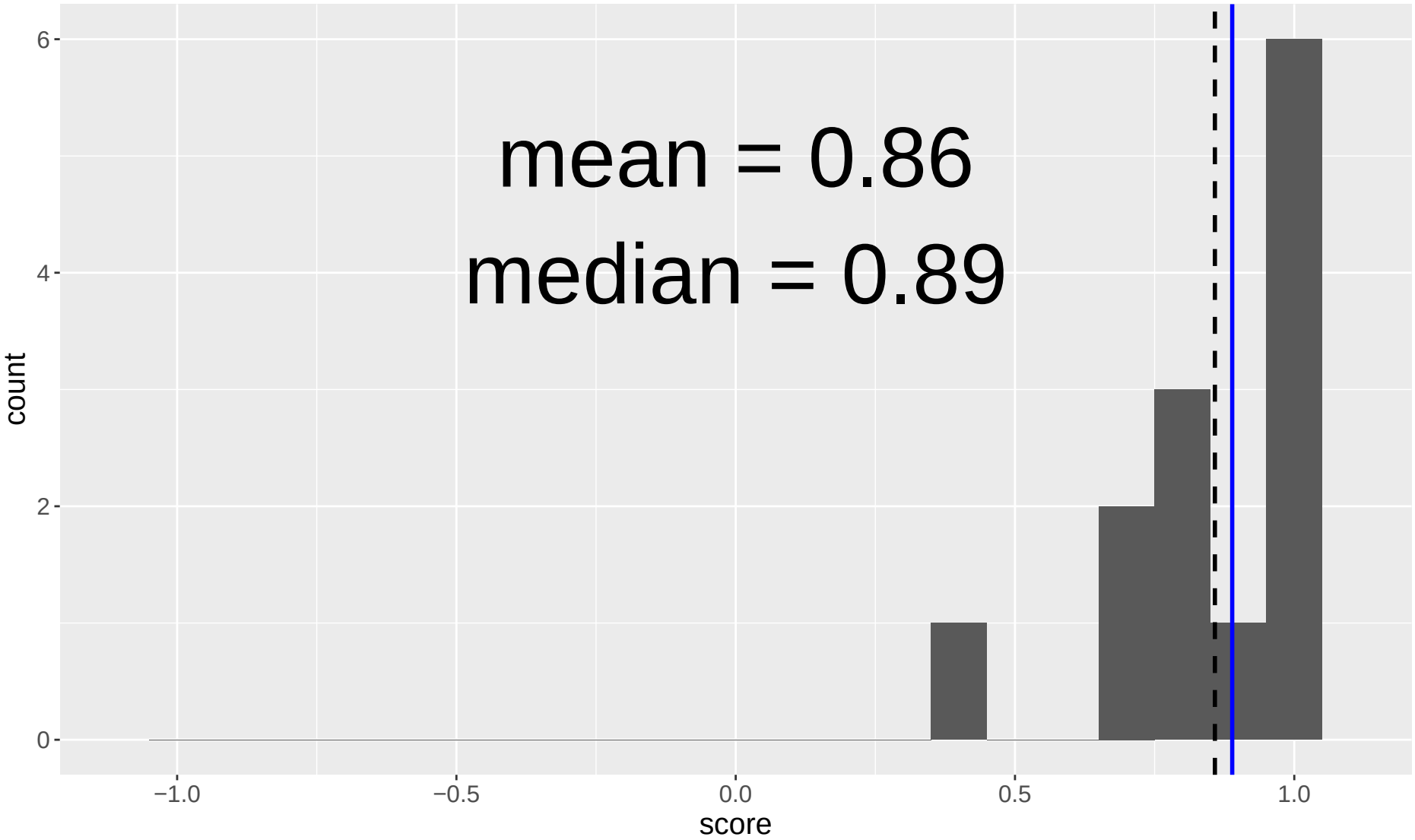


Figure 1: From Gelman and Nolan (2017)

Handedness Data Analysis



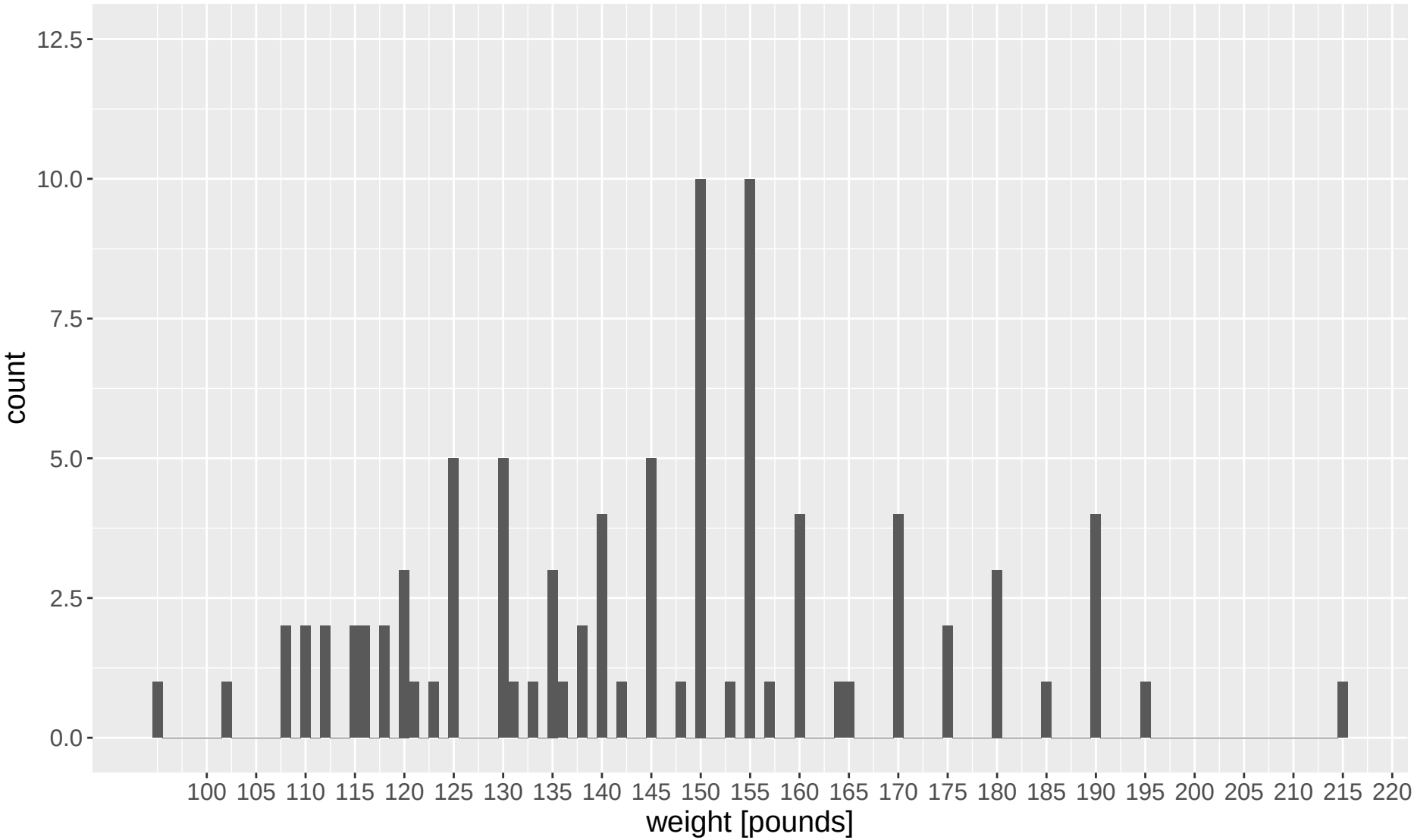
Handedness Data Analysis



Section 4

Reported Weights

Reported Weights



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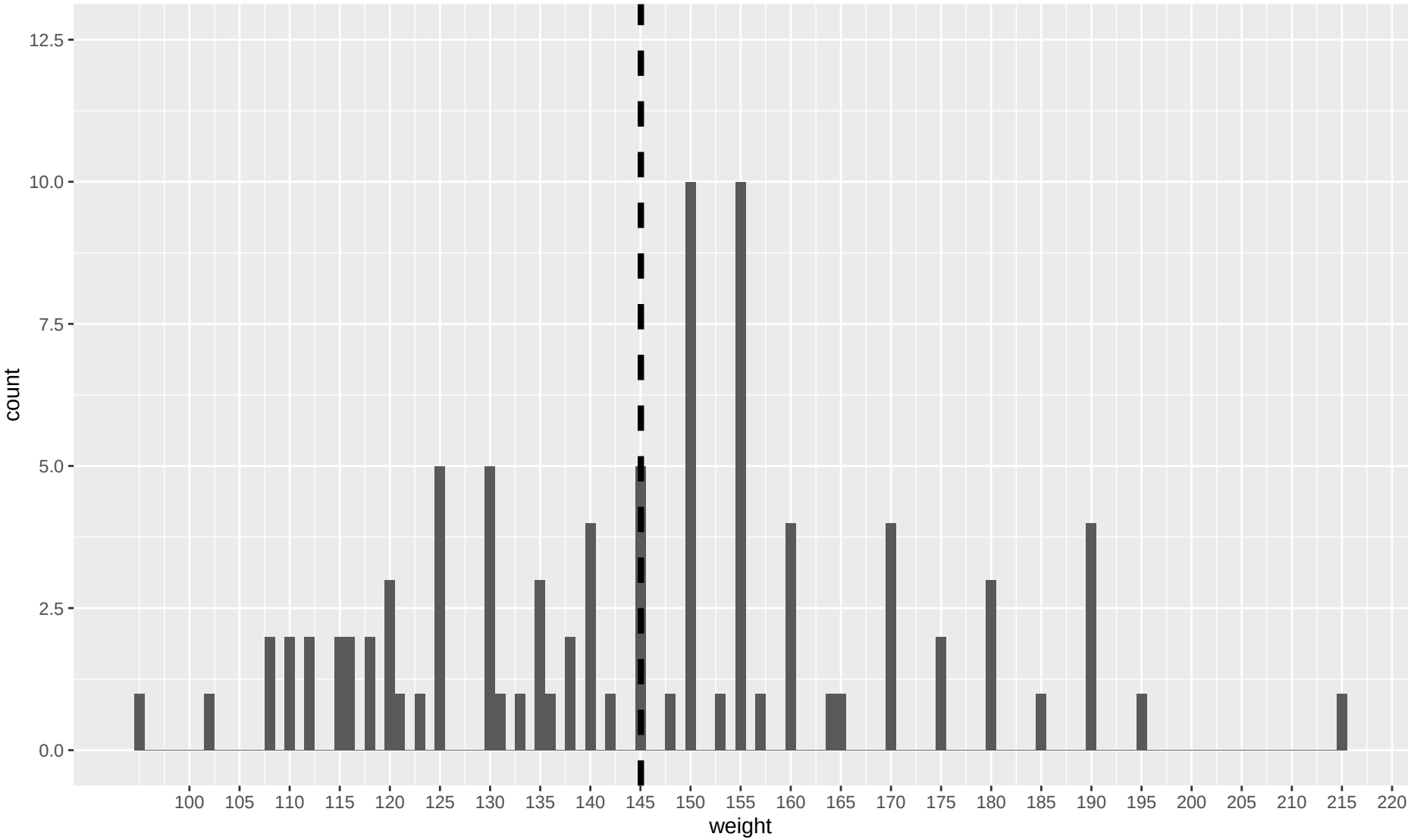

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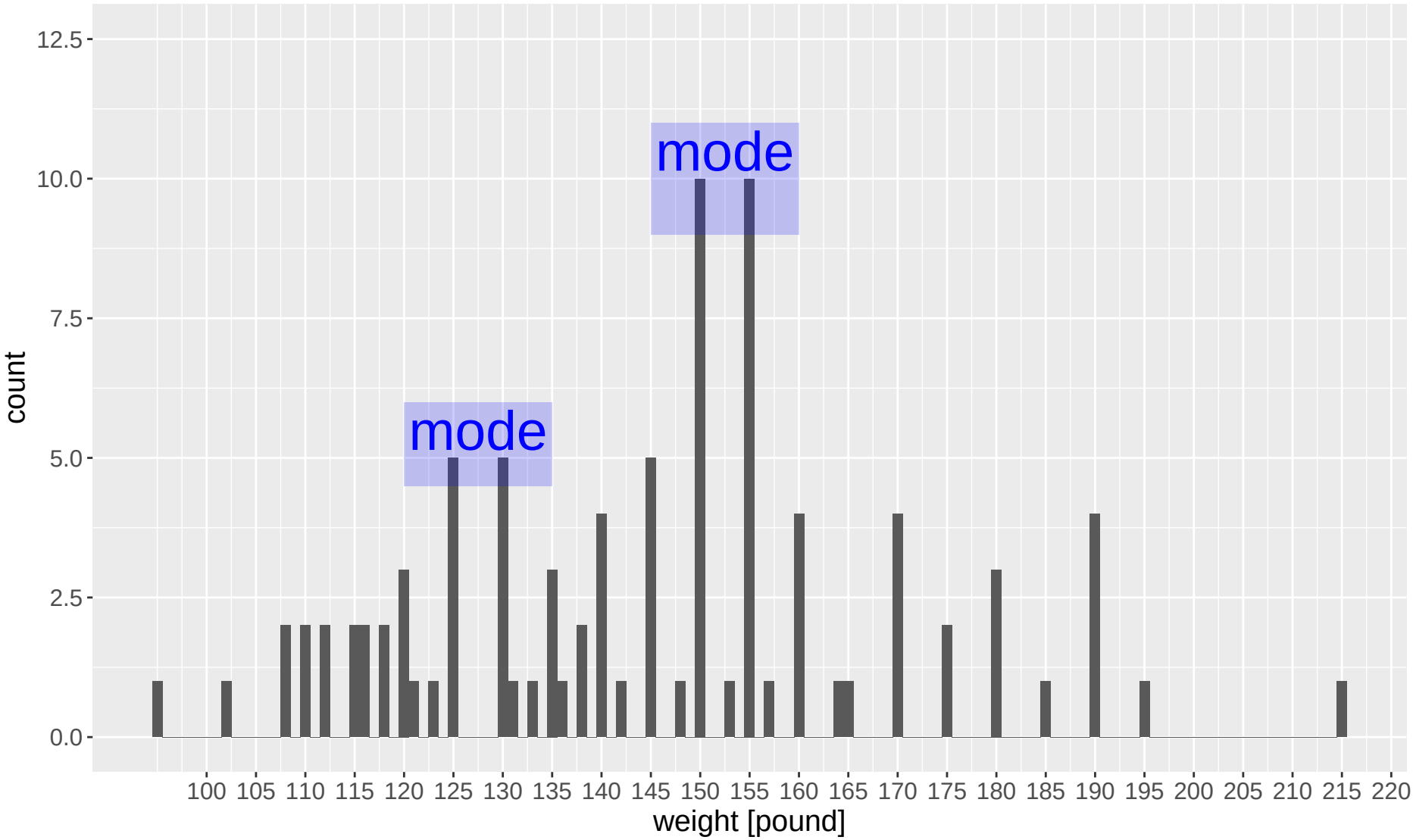
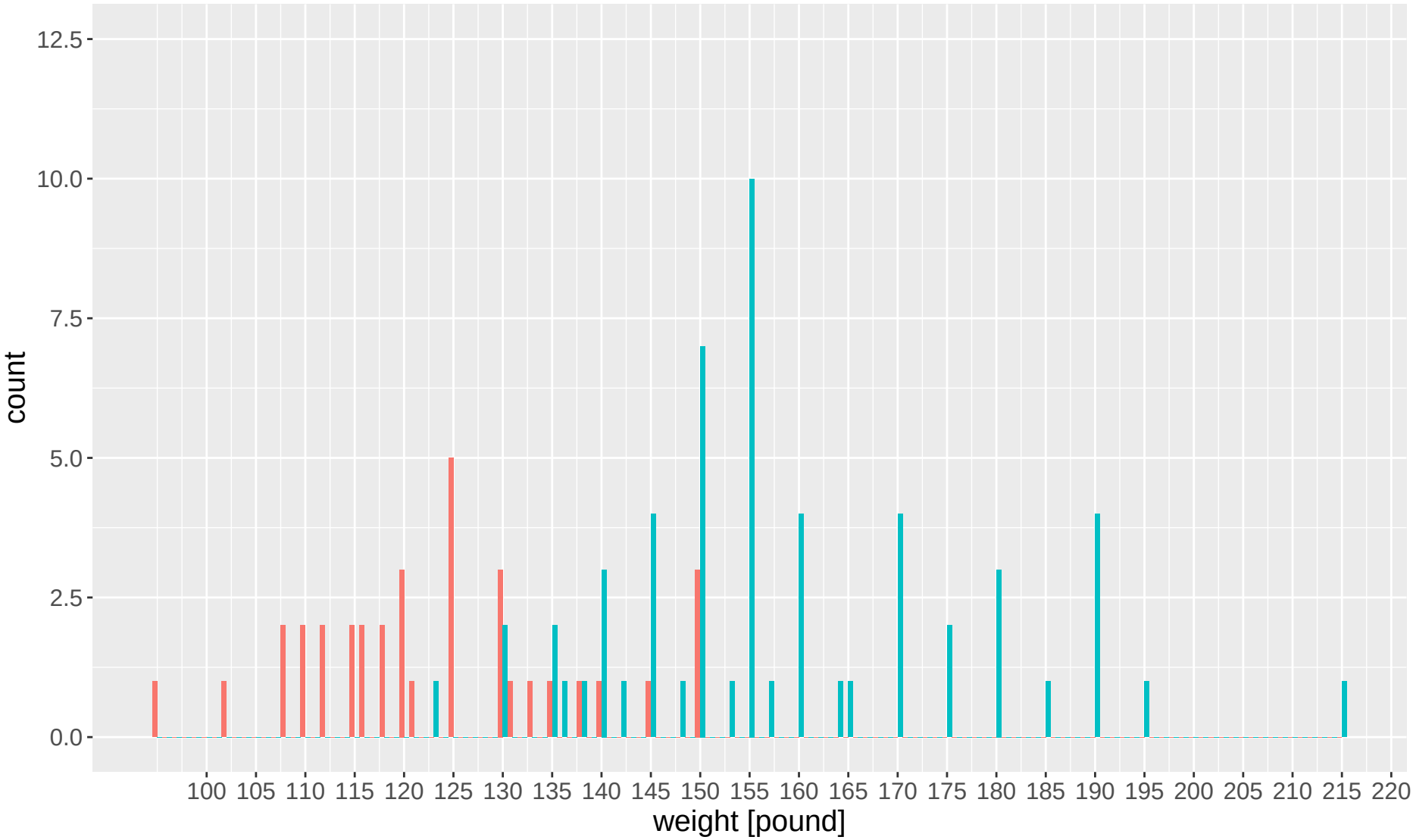


Reported Weights



Reported Weights

```
##
##   The decimal point is 1 digit(s) to the right of the |
##
##    9 | 5
##   10 | 288
##   11 | 0022556688
##   12 | 0001355555
##   13 | 0000013555688
##   14 | 00002555558
##   15 | 000000000035555555557
##   16 | 000045
##   17 | 000055
##   18 | 0005
##   19 | 00005
##   20 |
##   21 | 5
```

Section 5

Probability Mass/ Density Functions

Probability Mass/ Density Functions

- Continuous random variables can take any value with arbitrary precision (e.g. weight in kg)
- The probability of getting a very precise and specific value is very small
- Why? Because the interval for this value would be very small/ close to zero
- In the limit of infinitely small intervalls, the probability becomes zero
- This means the area under a point is zero, however, this doesn't mean that the point has zero value
- The distribution of continuous variables is therefore described by probability **density** functions

Probability Mass/ Density Functions

For discrete variables, the probability of X can be determined by summation over the probability **mass** function of x:

$$P(a \leq x \leq b) = \sum_{x:a \leq x \leq b} p(x)$$

For continuous variables, where intervals are infinitely small, the summation becomes an integral over the probability **density** function of x:

$$P(a \leq x \leq b) = \int_a^b p(x)$$

Probability Mass/ Density Functions

Properties of a probability density function (pdf) $p(x)$ are :

$$p(x) \geq 0$$

$$\int_{-\infty}^{\infty} p(x)\, dx = 1$$

$$P(x = a) = \int_{a}^{a} p(x)\, dx = 0$$

Probability Mass/ Density Functions

Probabilities are defined over intervalls

$$P\left(a \leq x \leq a+\delta\right)$$

where we define an interval $[a, a+\delta]$ of length δ and let $\delta \geq 0$ and *small*, then can approximate the probability as

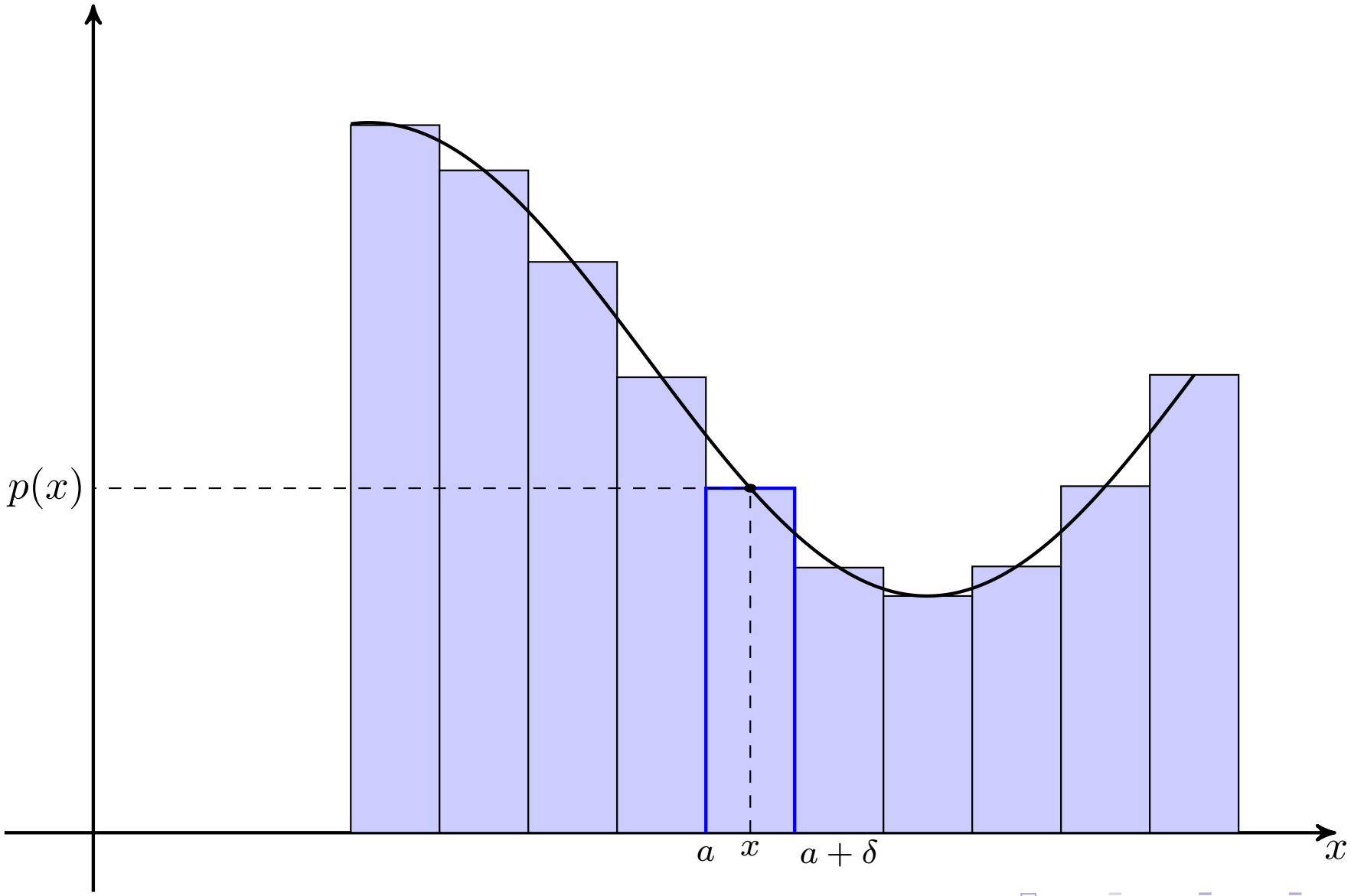
$$P\left(a \leq x \leq a+\delta\right) \approx p(a)\delta$$

and isolate $p\left(x\right)$ on the right-hand side

$$p\left(x\right) = P\left(a \leq x \leq a+\delta\right)/\delta$$

$p\left(x\right)$ is therefore called probability density: a probability per unit length

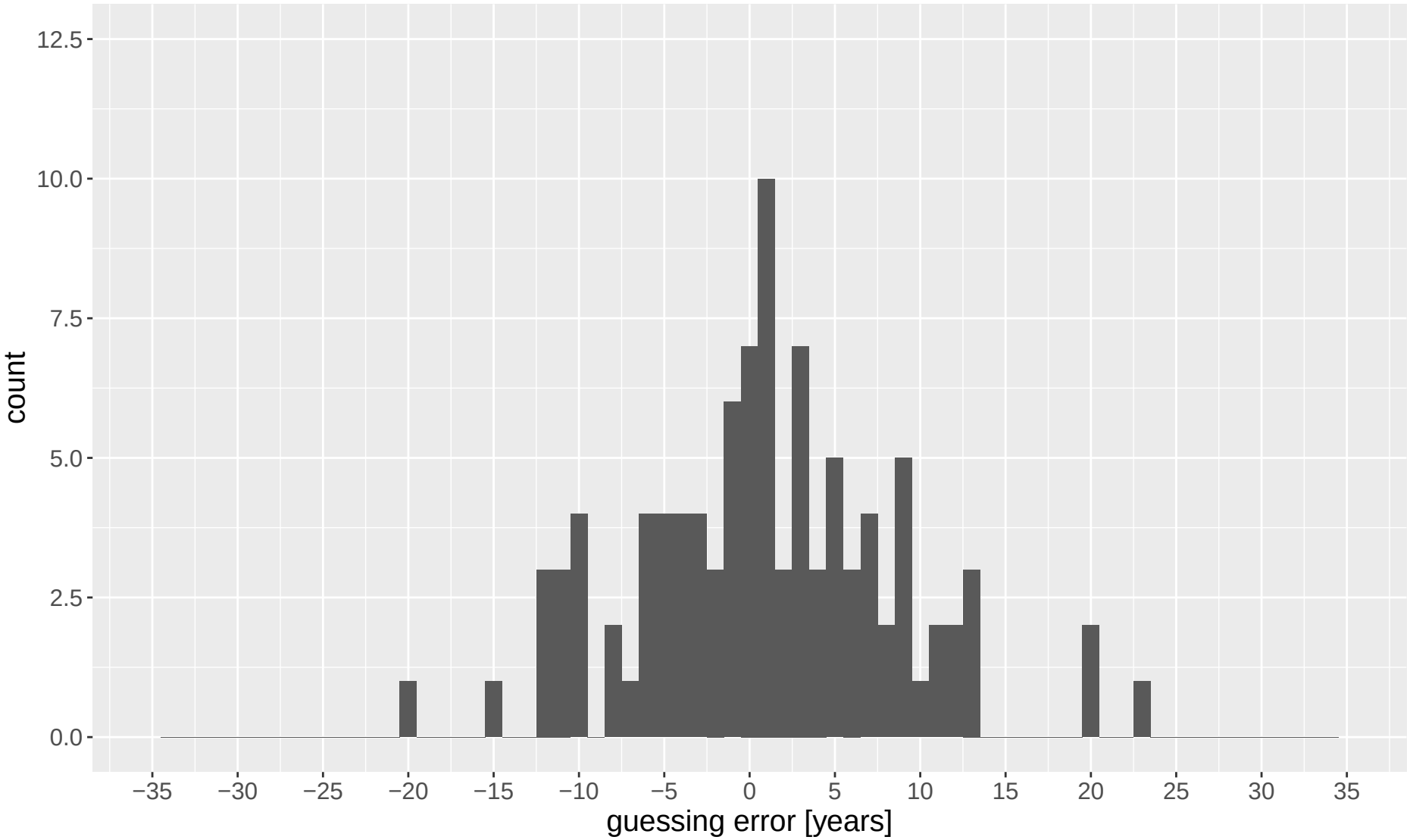
Probability Mass/ Density Functions



Section 6

Age guessing error

Age guessing error



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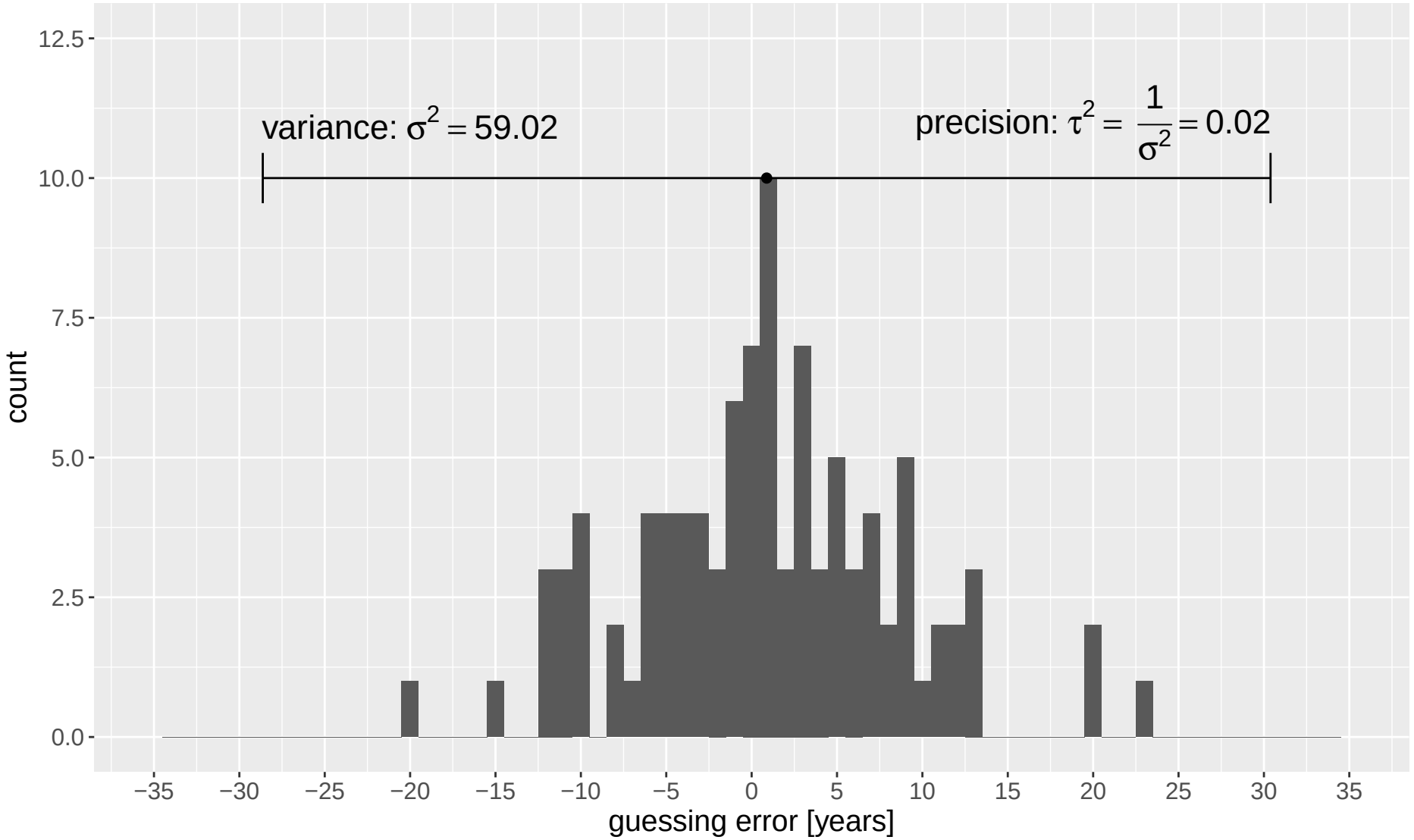
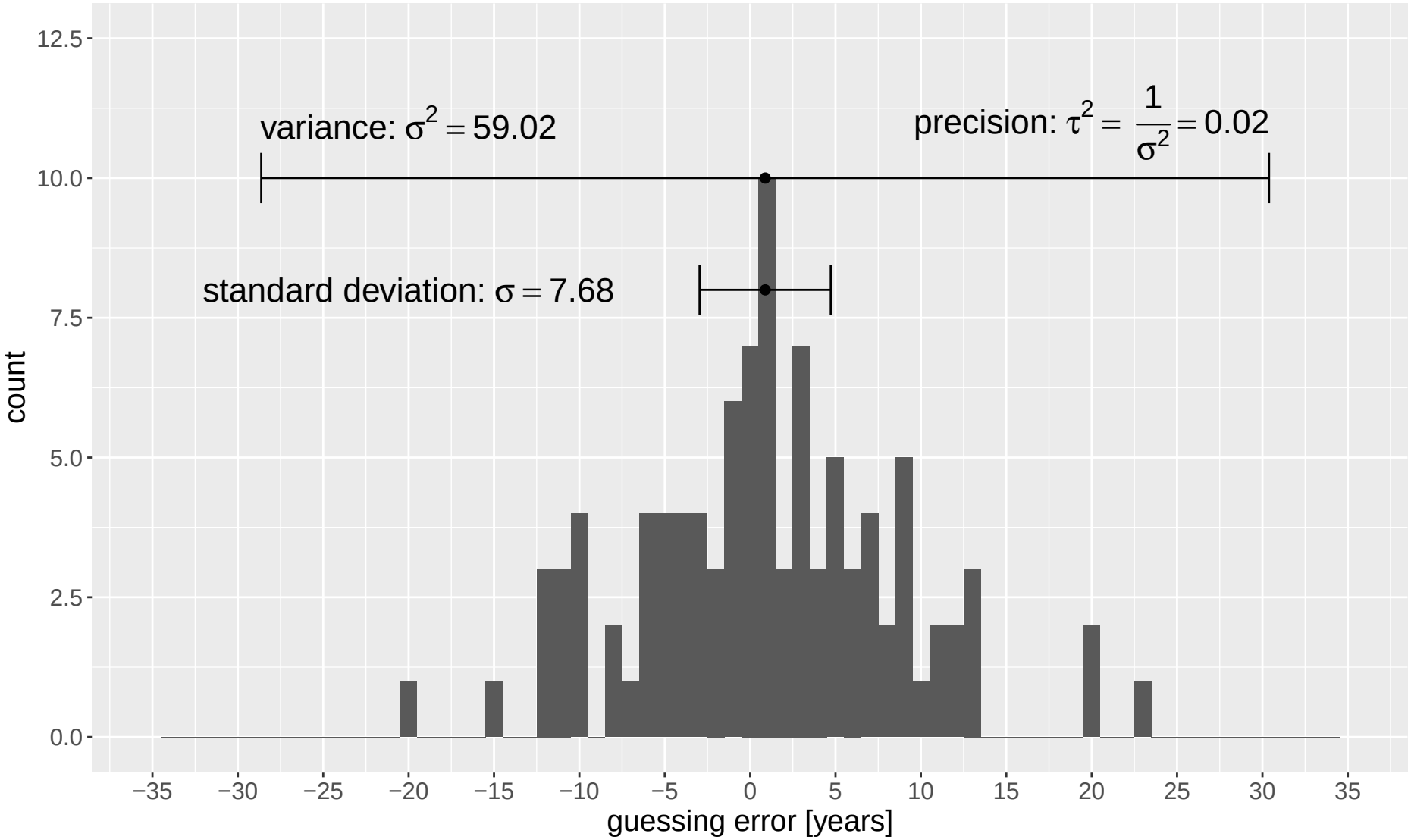


Age guessing error



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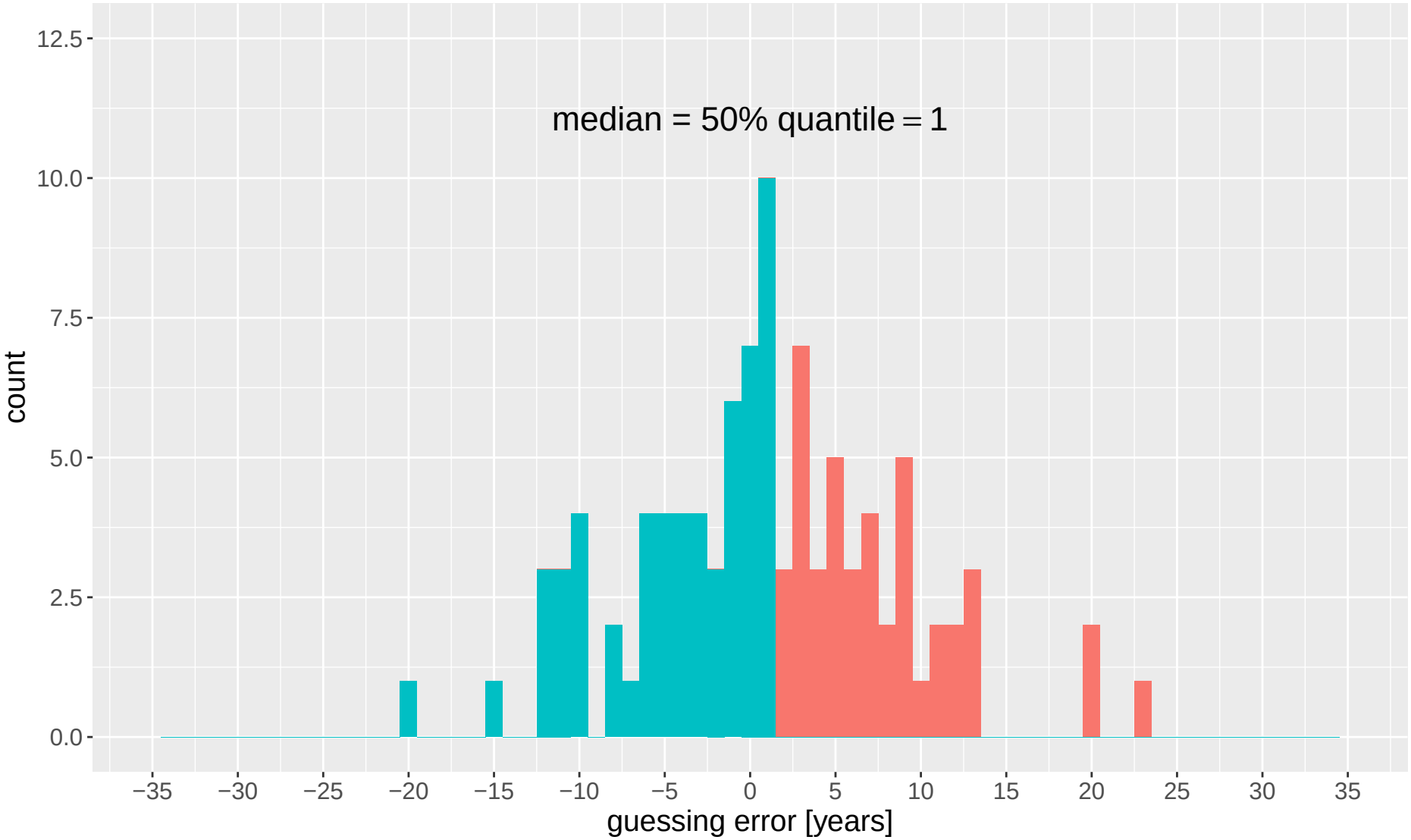

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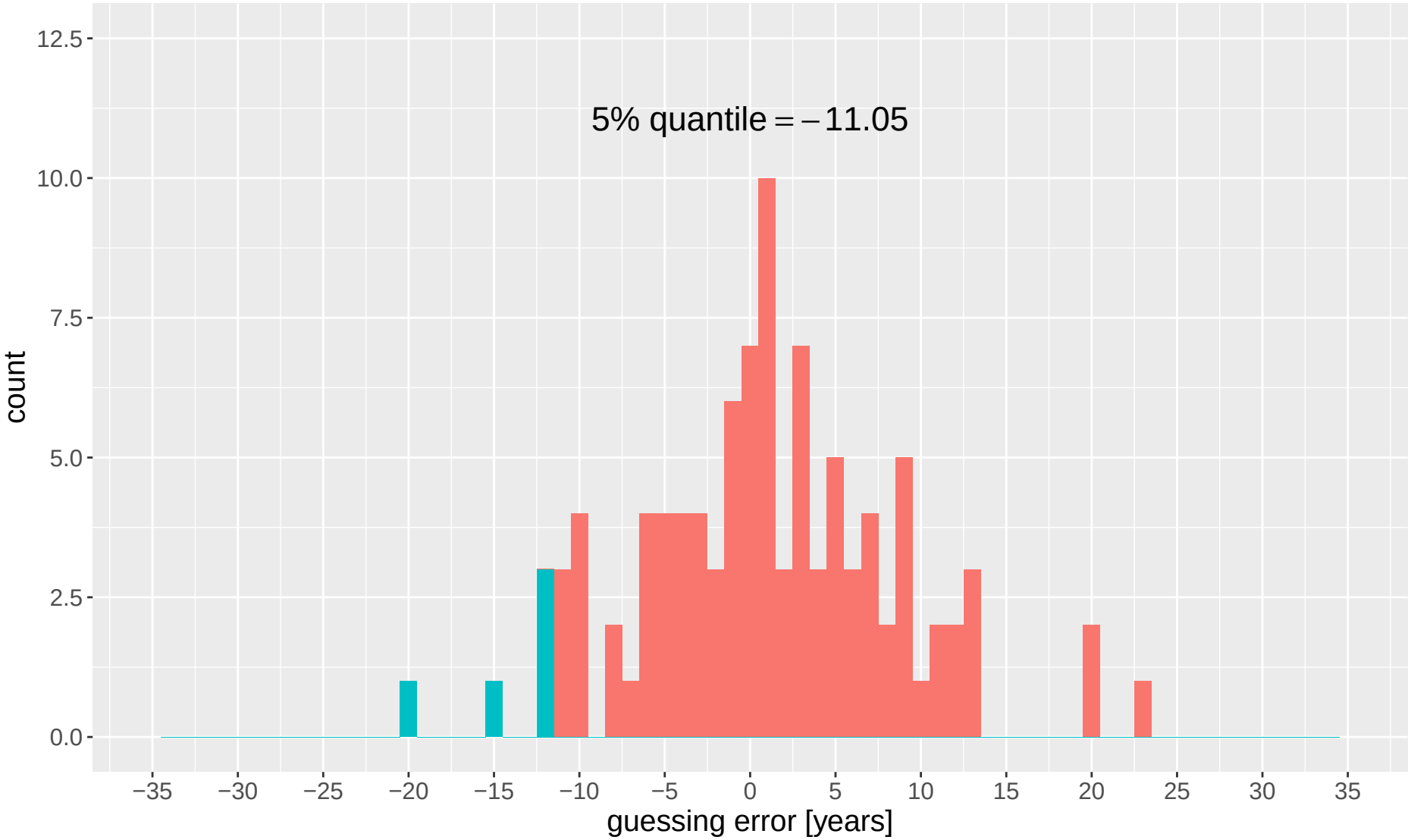

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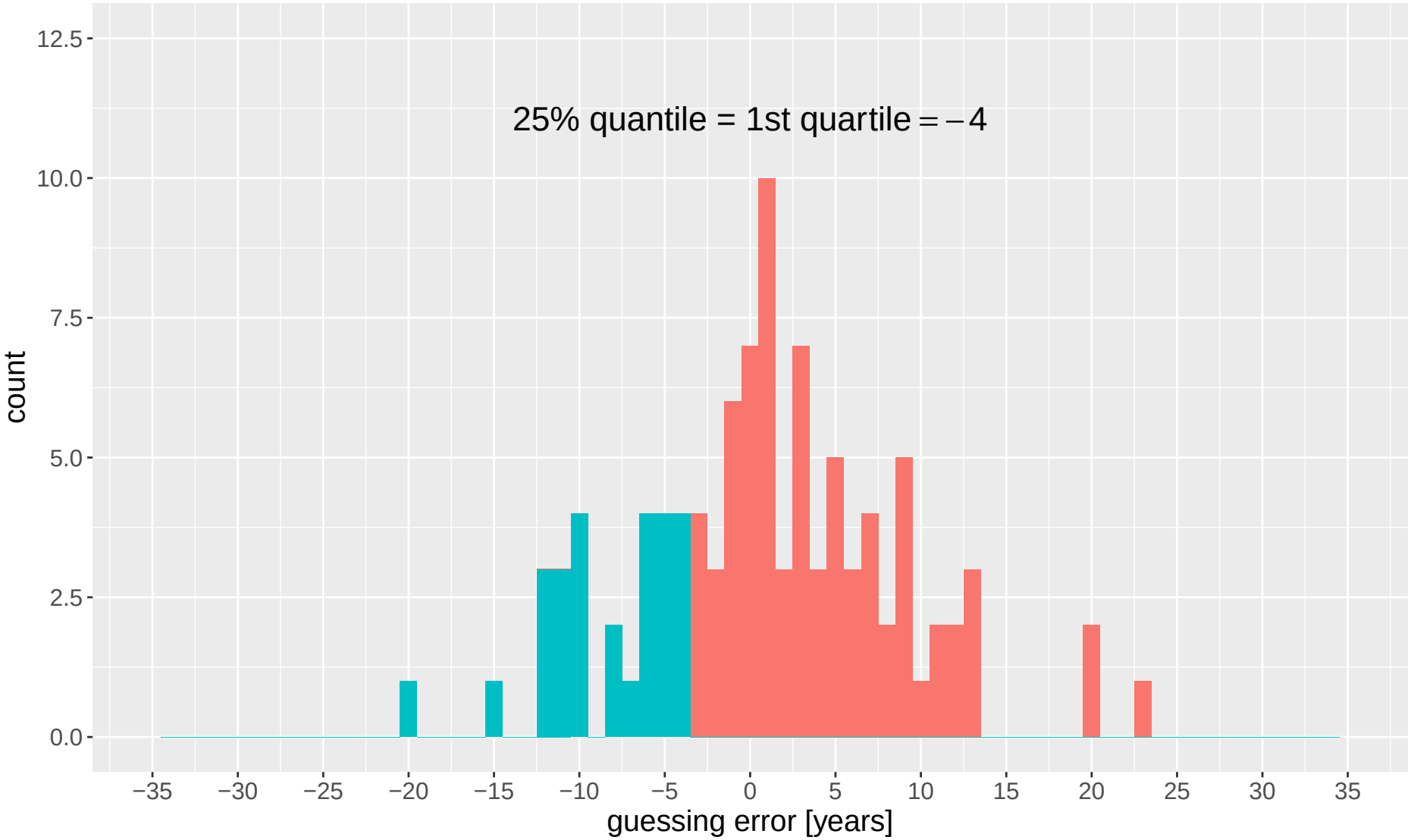

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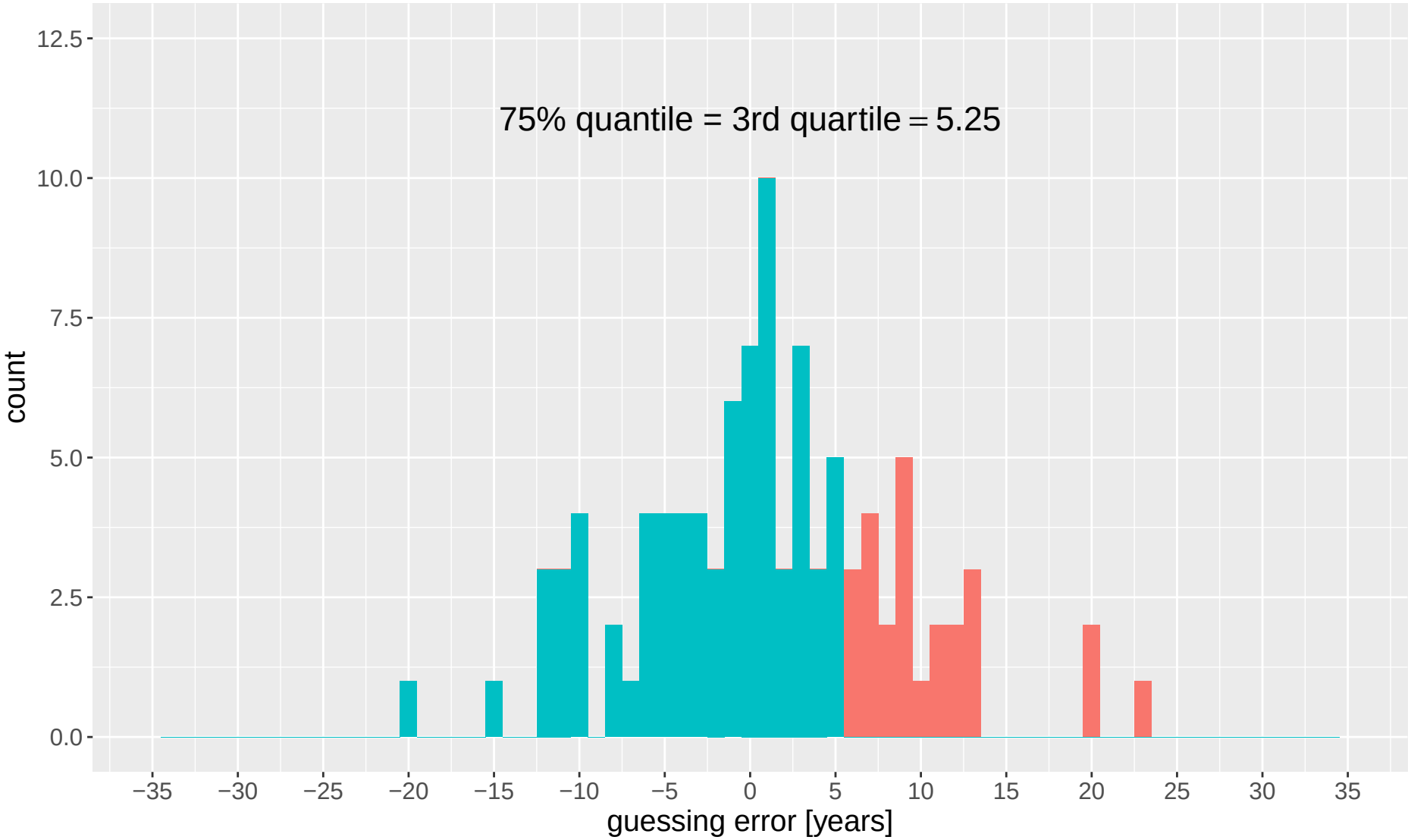

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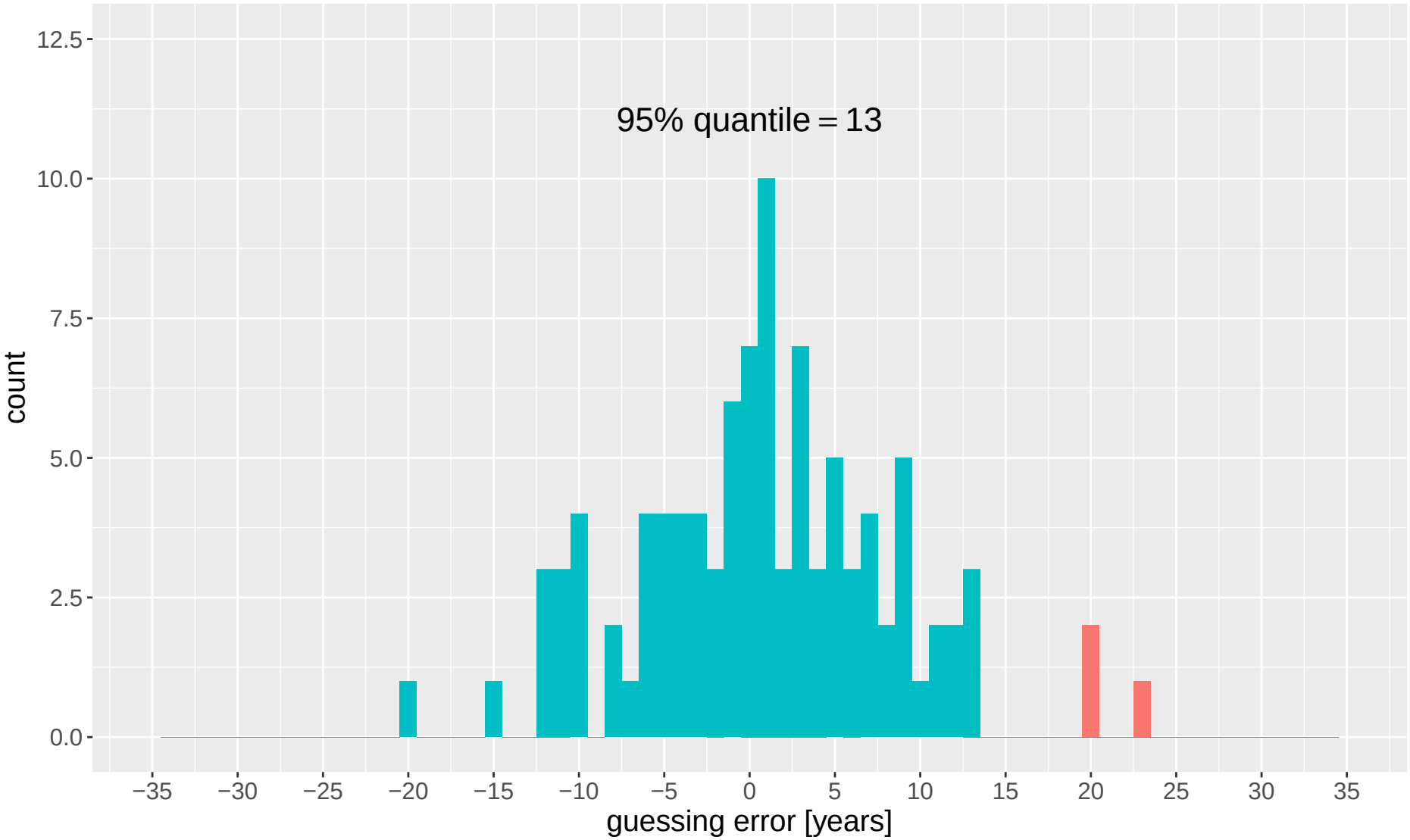

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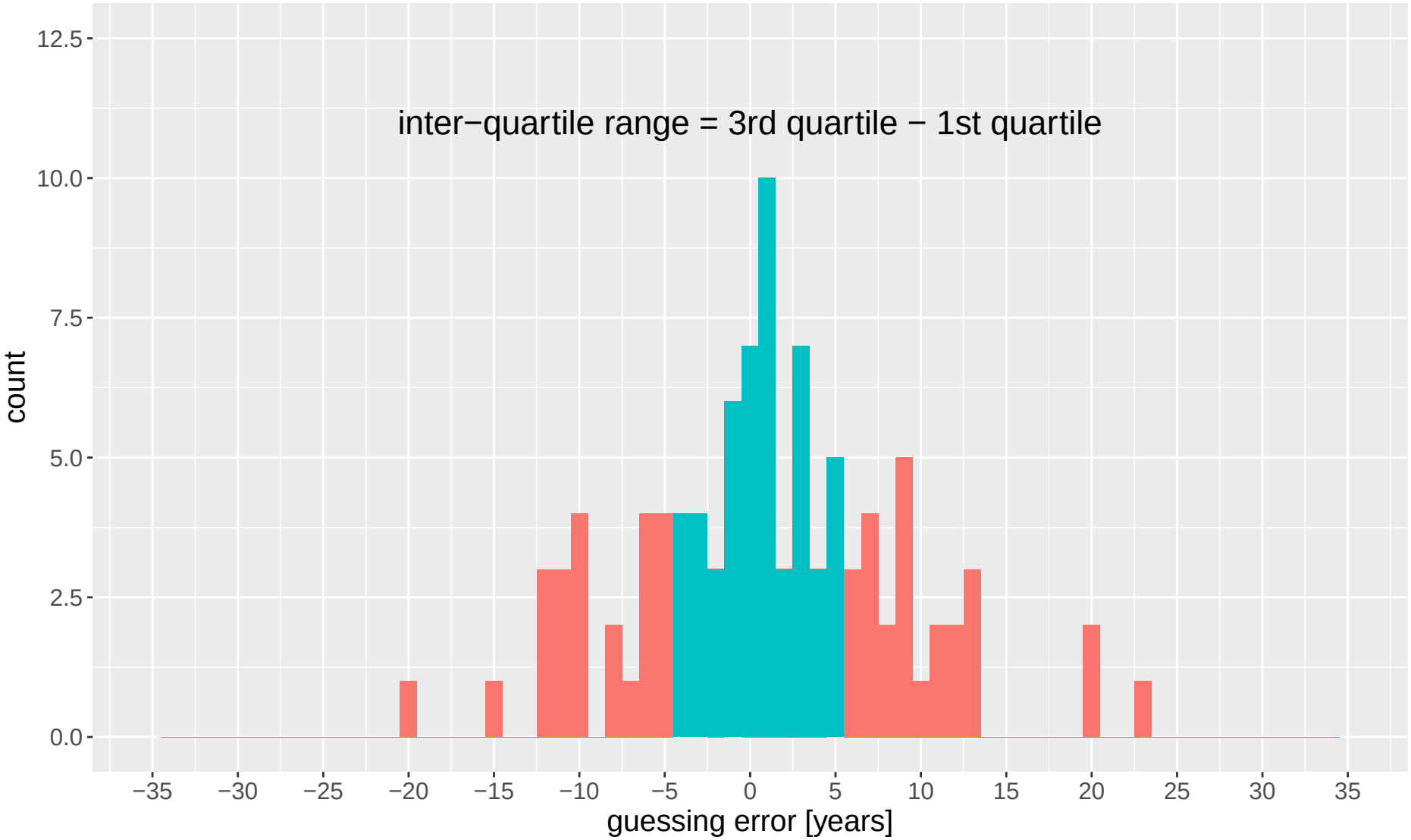

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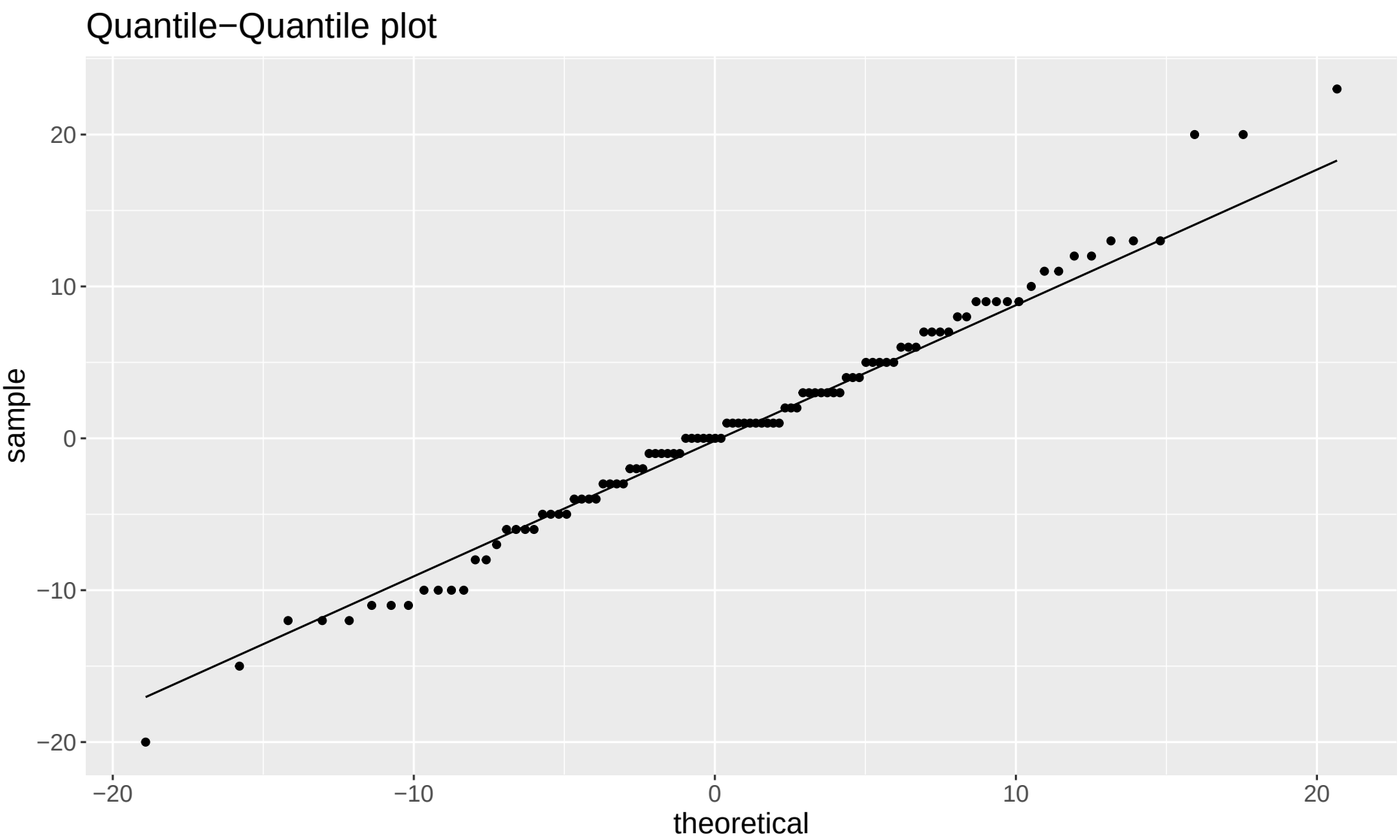

Section 7

Summary

Summary

- Data coding and formatting
- Numerical statistics:
  - scale/ spread:
    - variance/ precision (parameter)
    - standard deviation used as data summary
  - location:
    - mean (parameter and data summary)
    - median (data summary, more stable, less susceptible to extrem values)
    - mode (property of a distribution)
  - measures of distribution: quantiles/ percentiles/ quartiles
- Probability distribution:
  - probability **mass** distribtuion: discrete variables
  - probability **density** distribtuion: continuous distribution
  - properties of pmfs/ pdfs:
    - total area/ integral over domain: $\int_{-\infty}^{\infty} p(x)\, dx = 1$
    - pdfs: $P(x = a) = 0$

Andrew Gelman and Deborah Nolan. *Teaching statistics: A bag of tricks*. Oxford University Press, 2017.